

Reply to Comments on “Invariance of the tunneling method for dynamical black holes”

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We point out basic misunderstandings about quantum field theory and general relativity in the above Comments. In reply to a second comment on our first reply by the same author, we also identify precisely where the author’s original calculation goes wrong and correct it, yielding the same local Hawking temperature as obtained by the Hamilton-Jacobi method.

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In observance to the ArXiv policy about repeated submissions on the same subject, we attach (and replace) to a previous submission further considerations in reply to a second comment (arXiv:0909.3800) by the same author, which for some reason was not subjected to the same policy. For the sake of honesty, we repeat exact text of our first reply in the first section, postponing to a second section the new considerations referring to the second comment.

I. FIRST REPLY

Hawking radiation from black holes [1] is a well established prediction of quantum field theory on curved space-times, confirmed by multiple independent methods, see for example [2, 3, 4, 5, 6, 7]. We have recently generalized this prediction from static to dynamic black holes [8, 9, 10]. Conversely, an article of Pizzi [11] claims that Hawking radiation is a myth which has fooled almost everyone but himself. For some reason, he has chosen to write it in the form of a Comment on our article [10]. While we feel no need to defend anything in our article, here we point out briefly the main errors in the reasoning in [11].

1. The author states that “the action... along the classical light-like ray is... constant” and therefore “no imaginary part in the action can appear”. However, Hawking radiation is not a prediction of classical physics but of quantum field theory. In the WKB approximation, the action is not constant, but indeed rapidly varying.

2. The author states that an “infinitesimally small neighbourhood of the horizon... can be covered by Minkowski coordinates”. This is incorrect. The correct statement is that connection coefficients, being first derivatives of the metric, can be set to zero at a point. Surface gravity and the corresponding temperature are curvature invariants, which involve second derivatives of the metric and cannot be set to zero at a point.

3. There is a confusion of partial derivatives in the author’s equation (3). He appears to be solving the null geodesic equation rather than the Hamilton-Jacobi equation.

4. The author’s procedure for dealing with the pole in the action is inequivalent to the standard one, namely the Feynman $i\epsilon$ procedure or something equivalent, which corresponds to the desired physical boundary conditions. It has not been justified and is used by no other author as far as we are aware.

II. SECOND REPLY

1. The author states in [12] that “to affirm that because of quantum theory we will have in the principal WKB approximation an action of essentially different character is nonsense”. This is untrue. When going from classical physics to quantum field theory, qualitatively new features occur which cannot be found in the classical limit.

2. The author has not answered our point 2, that a fundamental misunderstanding about general relativity was the basis of one of his two arguments.

3. The author states in [12] that “the equation (3)... is a trivial equation which in no way can contain any confusion”. Here we explain the confusion. The equation reads

$$I = \int (\partial_r I dr + \partial_v I dv) \quad (1)$$

$$= \int \left(\partial_r I - \frac{1}{2} \frac{dv}{dr} e^\Psi C \partial_r I \right) dr \quad (2)$$

where

$$\frac{dv}{dr} = \frac{2e^{-\Psi}}{C} \quad (3)$$

was obtained by solving the null geodesic equation. This derivative is infinite at the horizon $C = 0$, so the above manipulation is invalid. Also

$$\partial_r I = - \frac{2e^{-\Psi} \partial_v I}{C} \quad (4)$$

is infinite at the horizon, as the angular frequency

$$\omega = e^{-\Psi} \partial_v I \quad (5)$$

is finite. So (2) is meaningless, infinity minus infinity.

Directly substituting (4), (5) into (1) yields

$$I = \int e^{\Psi} \omega dv - \int \frac{2\omega}{C} dr \quad (6)$$

which is the same expression as obtained by the Hamilton-Jacobi method [9, 10]. The first term is finite, while the second has a pole at $C = 0$. This is the famous pole in the action.

4. Therefore the author's claim in [12] that there "are no poles in the integrand of the action" is false.

5. The author suggests in [12] "to focus firstly on the simpler case of the Schwarzschild black hole, where only very well known formulas are used and any eventual mistake is easy to be discovered". Indeed, in this case it is easy to see that dr/dv vanishes along the horizon, where r is constant.

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